

# RESTRICTED 內部文件

香港考試局  
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一九八八年香港中學會考  
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數學  
Mathematics

評卷參考  
Marking Scheme

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## 請在學校任教之閱卷員特別留意

本評卷參考並非標準答案，故極不宜  
落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴  
予拒絕。閱卷員在任何情況下披露本  
評卷參考內容，均有違閱卷員守則及  
「一九七七年香港考試局法例」。

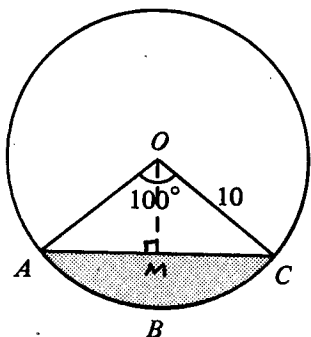
## Special Notes for Teacher Markers

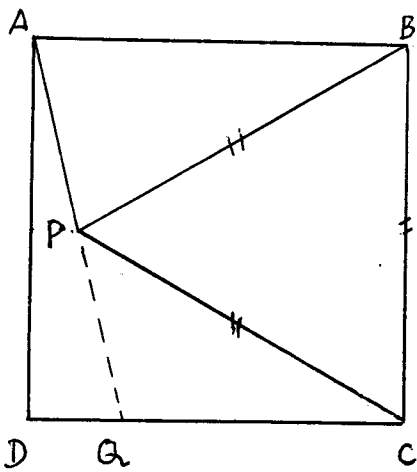
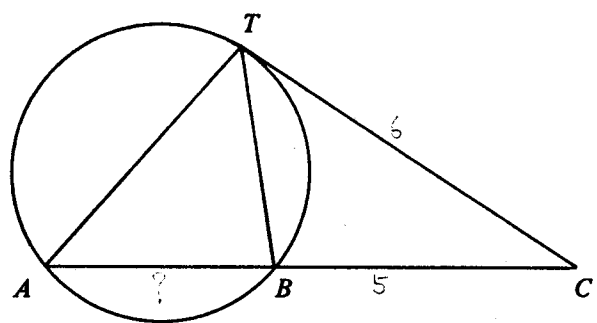
It is highly undesirable that this  
marking scheme should fall into the  
hands of students. They are likely  
to regard it as a set of model  
answers, which it certainly is not.

Markers should therefore resist  
pleas from their students to have  
access to this document. Making it  
available would constitute mis-  
conduct on the part of the marker  
and is, moreover in breach of the  
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Solutions	Marks	Remarks
1. $a^2 - a - 6 = (a + 2)(a - 3)$ $a^3 + 8 = (a + 2)(a^2 - 2a + 4)$ Their L.C.M. = $(a + 2)(a - 3)(a^2 - 2a + 4)$ $(= a^4 - 3a^2 + 8a - 24)$	<div>any 1 correct ..... all correct the others correct</div> <div>2A+1A 1M+1A <hr/>5</div>	2A for first correct part Both exp. must first be factorized. <div>PP-1 at most 3 per paper at most 1 per question at most 1 for the same type of qp.</div>
2. (a) $\frac{\sin(180^\circ - \theta)}{\sin(90^\circ + \theta)} = \frac{\sin\theta}{\cos\theta}$ must be shown..... $= \tan\theta$ (b) $\sin^2(\pi - \theta) + \sin^2(\frac{3\pi}{2} + \theta)$ $= \sin^2\theta + \cos^2\theta$ $\sin\phi - \cos\phi \dots 0A$ $= 1$ .....	<div>1A 1A 1A  1A <hr/>1A 5</div>	EXCESS For $\sin(\frac{3\pi}{2} + \theta) = -\cos\theta$
3. $2x^2 \geq 5x$ $2x^2 - 5x \geq 0$ $x(2x - 5) \geq 0$ .....	<div>1A 1A  3A <hr/>5</div>	Withhold 1 mark if '=' omitted. If solved by equation, no marks awarded unless answer correct. <div>Optional any 1 part without = , withhold 1 mark.</div> <div>For <math>x \leq 0, x \geq \frac{5}{2}</math> , 2 <math>x \leq 0</math> and <math>x \geq \frac{5}{2}</math> 1</div>
4. (a) If $9x^2 - (k + 1)x + 1 = 0$ has equal roots, $(k + 1)^2 - 36 = 0$ .....	<div>1A 1A 1A 1A 1M  <hr/>1A 6</div>	Alt. Solution: $(k+1)^2 - 36 = 0$ 1A $k + 1 = \pm 6$ 1A+1A $k = 5$ or $-7$ 1A $k+1 = 6$ 1A only Sub. For negative value of k L.S. = $(3x + 1)^2$ $x = -\frac{1}{3}$

Solutions	Marks	Remarks
5. (a) Area of OABC = $\pi 10^2 \times \frac{100^\circ}{360^\circ}$ ..... = 87.27 (corr. to 2 d.p.) (or 87.28)	1M 1A	
(b) Area of $\triangle OAC = \frac{1}{2} \times 10 \times 10 \times \sin 100^\circ$ = 49.24 (corr. to 2 d.p.)	1M 1A	$\Delta = \frac{1}{2} AC \times OM$ $= \frac{1}{2} \times 15.3209 \times 6.4279$ ... 1M = 49.24       ... 1A
(c) Area of minor segment ABC = 87.27 - 49.24 ..... = 38.03 (corr. to 2 d.p.) (or 38.04)	1M <u>1A</u> <u>6</u>	
6. $\log 2 = r$ , $\log 3 = s$ . (a) $\log 18 = \log 2 \times 3^2$ = $\log 2 + \log 3^2$ ) ..... = $\log 2 + 2\log 3$ ) ..... = $r + 2s$ .....	1A 1M 1A	For $18 = 2 \times 3^2$ ) $\log ab = \log a + \log b$ or ) $\log a^2 = 2\log a$
(b) $\log 15 = \log 3 \times 5$ = $\log 3 + \log 5$ = $\log 3 + \log \frac{10}{2}$ ) A ..... = $\log 3 + \log 10 - \log 2$ = $1 - r + s$ .....	1A 1A <u>1A</u> <u>6</u>	For $5 = \frac{10}{2}$ or $15 = \frac{30}{2}$
7. (a) The coordinates of the centre are given by $x = -(-\frac{4}{2})$ , $y = -\frac{10}{2}$ ..... i.e. $x = 2$ , $y = -5$	1M 1A	$(x-2)^2 + (y+5)^2 = 25$ k++
(b) As C touches the y-axis, its radius = 2 ..... $4 + 25 - k = 2^2$ $k = 25$	1M+1A 1M 1A <u>6</u>	OR Subs. (0, -5) 1M $25 - 50 + k = 0$ $k = 25$ 1A $r = \sqrt{4 + 25 - 25}$ 1M = 2 ..... 1A OR Put $x = 0$ , $y^2 + 10y + k = 0$ has equal roots. 1M $100 - 4k = 0$ $k = 25$ 1A $r = \text{etc.}$

Solutions		Marks	Remarks
8. (a) (i)	 <p>(ii) Since <math>\triangle PBC</math> is equilateral, <math>\angle PBC = 60^\circ</math></p> <p><math>\angle ABP = 90^\circ - 60^\circ = 30^\circ</math> .....</p> <p>As <math>BA = BP</math>, <math>\angle PAB = \frac{1}{2}(180^\circ - 30^\circ)</math></p> <p><math>= 75^\circ</math></p> <p>Since <math>AB \parallel DC</math>, <math>\angle PQC = 180^\circ - 75^\circ</math></p> <p><math>= 105^\circ</math></p>	<p>1</p> <p>1</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>7</p>	<p>ABCD in order</p> <p>For P</p> <p>For Q (between D, C)</p> <p>Follow through even if diagram not accurate</p> <p>or equivalent</p> <p>OR</p> <p><math>\angle PAD = 15^\circ</math></p> <p><math>\angle PQC = 90^\circ + 15^\circ = 105^\circ</math></p> <p>1M</p> <p>1A</p>
(b) (i)	<p><math>\triangle TCB</math> is similar to <math>\triangle ACT</math> because</p> <p><math>\angle C</math> is common.</p> <p><math>\angle BTC = \angle BAT</math> (angle in alternate segment)</p> <p><math>\angle T</math> no mark</p> <p><math>\triangle TCB \sim \triangle ACT</math> (A.A.A.)</p> <p>(ii) <math>\frac{AC}{CT} = \frac{CT}{BC}</math> .....</p> <p><math>AC = \frac{6^2}{5} = 7.2</math> correct substitution</p> <p><math>\therefore AB = 7.2 - 5</math></p> <p><math>= 2.2 (= \frac{11}{5})</math> .....</p>	<p>1</p> <p>1</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>5</p>	<p><math>\approx</math></p> <p><math>\cong</math></p> <p>PP-1</p> <p>Indication of 2 pairs of equal angles. Withheld if proving congruence.</p> <p>Follow through even if (b)(i) wrong.</p>
			

Solutions	Marks	Remarks
9. (a) Between 100 and 999, the smallest multiple of 7 is 105, the largest is 994.	1A <u>1A</u> 2	
(b) The number of multiples is $\frac{994 - 105}{7} + 1$ must be correct. = 128  The sum of these multiples = $105 + 112 + \dots + 994$ = $\frac{128}{2} [105 + 994]$ ..... = 70336	2M 1A    2M <u>1A</u> 6	OR $994 = 105 + (n-1) \times 7$
(c) The sum of all positive 3-digit integers = $100 + 101 + \dots + 999$ = $\frac{900}{2} [100 + 999]$ } or all correct = 494,550 .....  The required sum = $494,550 - 70,336$ = 424,214	1 <del>1A</del>  1A 1M <u>1A</u> 4	

Solutions	Marks	Remarks
<p>10. (a) Let <math>y = k_1x + k_2x^2</math>, where <math>k_1</math> and <math>k_2</math> are constants.</p> <p>Putting <math>x = 1, y = -5; x = 2, y = -8</math>, we have</p> $k_1 + k_2 = -5 \dots\dots\dots$ $2k_1 + 4k_2 = -8$ <p>Solving, <math>k_1 = -6, k_2 = 1</math></p> $\therefore y = -6x + x^2$ <p>Putting <math>x = 6</math>, we have <math>y = 0</math>.</p>	<p>2</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1A+1A</p> <p>1A</p> <p>8</p> <p>1M</p> <p>1A</p> <p>1M+1A</p> <p>4</p>	<p>For <math>y=kx+kx^2</math> or <math>y = kx+x^2</math></p> <p>or <math>y = x+kx^2 \dots\dots\dots 1</math></p> <p><math>y = x + x^2</math> no marks</p> <p>no marks <math>\begin{cases} y=k_1x \\ y=k_2x^2 \end{cases}</math></p> <p>Equality must hold.</p> <p><math>y = (x+3)^2 - 9</math> OA</p> <p>least value of <math>y</math> is <math>-9</math> 1M OA</p>
<p>(b) <math>y = -6x + x^2 = (x^2 - 6x + 9) - 9</math></p> $= (x - 3)^2 - 9 \dots\dots\dots$ <p>When <math>x = 3</math>, the value of <math>y</math> is least and the least value is <math>-9</math>.</p>		
<p>11. (a) From the curve,</p> <p>(i) the median is 70 marks.</p> <p>(ii) the 1st quartile is 50 marks. )</p> <p>the 3rd quartile is 86 marks. ) <math>\dots\dots</math></p> <p><math>\therefore</math> the interquartile range <math>= 86 - 50</math></p> $= 36 \text{ marks}$	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>4</p> <p>1A</p>	<p>for either</p>
<p>(b) (i) From the curve, the number of prize-winners <math>= 60</math>.</p> <p>(ii) The probability that the student is a prize-winner <math>= \frac{60}{600}</math> (<math>= \frac{1}{10}</math>).</p> <p>(iii)(1) The probability that both are prize-winners is <math>\frac{60}{600} \times \frac{59}{599} = \frac{59}{5990}</math> (<math>= 0.01</math>)</p> <p>(2) The probability that both are not prize-winners <math>= \frac{540}{600} \times \frac{539}{599} (= \frac{4851}{5990})</math> (<math>= 0.81</math>)</p> <p><math>\therefore</math> the probability that at least one is a prize-winner <math>= 1 - \frac{4851}{5990}</math></p> $= \frac{1139}{5990} (= 0.19)$	<p>1A</p> <p>1M+1A</p> <p>1M+1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>8</p>	<p>Accept <math>\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}</math></p> <p>1M for product rule</p> <p>Accept <math>\frac{9}{10} \times \frac{9}{10}</math></p> <p>OR</p> <p><math>\frac{9}{10} \times \frac{60}{599} + \frac{1}{10} \times \frac{540}{599}</math></p> <p><math>+ \frac{1}{10} \times \frac{59}{599}</math> 1M+1A</p> <p><math>= \frac{1139}{5990} \dots\dots\dots 1A</math></p>

## Solutions

## Marks

## Remarks

12. (a)  $L_3$  is given by  $\frac{x}{3} + \frac{y}{4} = 1$

i.e.  $4x + 3y = 12$  .....

$$\frac{y-4}{x} = -\frac{4}{3} \quad \leftarrow \text{wrong slope}$$

$$\frac{y}{4} + \frac{x}{3} = 1$$

(b) The three constraints are  $y \leq 4$

$x \leq 3$

$4x + 3y \geq 12$  .....

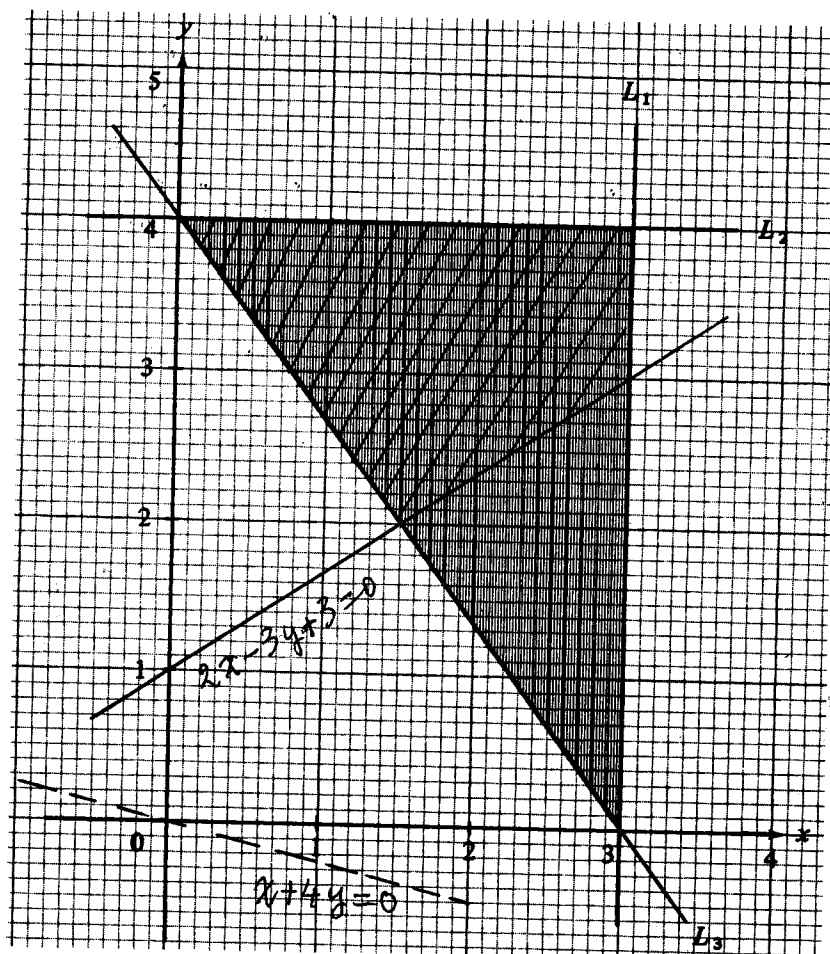
(c) The line  $x + 4y = c$  drawn in the diagram.

From the diagram,  $P$  is greatest when  $x = 3$ ,  
 $y = 4$  and least when  $x = 3$ ,  $y = 0$ .

only answer  
2A

The greatest value of  $P = 19$ ,

the least value = 3. ....



(d) The line  $2x - 3y + 3 = 0$  drawn in the diagram.

The shaded region.

$P$  is least when  $x = \frac{3}{2}$ ,  $y = 2$ .

The least value =  $\frac{19}{2}$  (= 9.5) .....

1M

or 2-pt form, etc.

1A

Must be in this form.

2

1A

Withhold 1 mark if '=' omitted.

1A

1A

or  $4x + 3y - 12 \geq 0$ .

3

1M+1A

For 1A  
 Drop of 2-3 verticle  
 units for 10 hori-  
 zontal units.  
 OR Testing any vertices

1A

At (3, 0),  $P = 3$ .

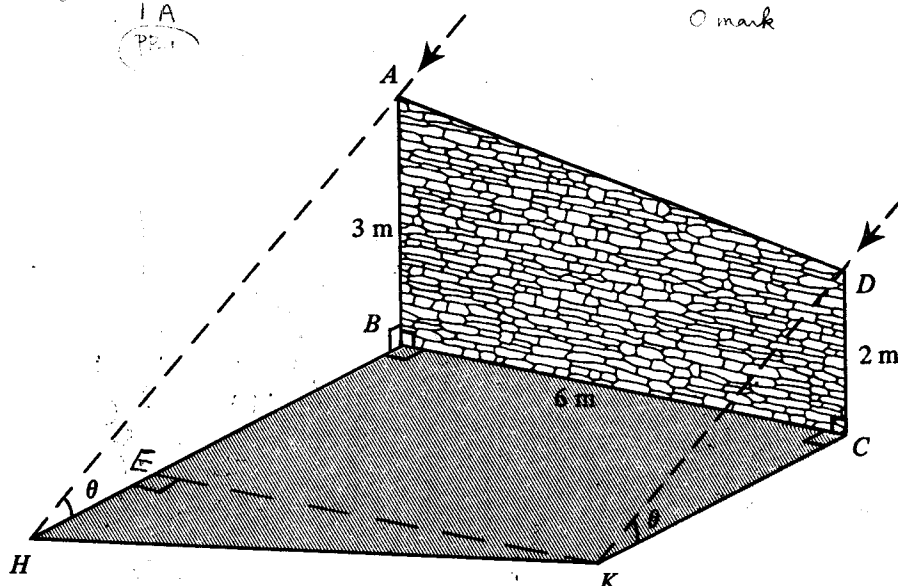
1A

At (0, 4),  $P = 16$ .At (3, 4),  $P = 19$ . 1A

4

test 2 points only 1M

Solutions		Marks	Remarks
13. (a)	$\frac{AB}{HB} = \tan\theta$ $\frac{HB}{\tan\theta} = 3 \text{ m}$ $\frac{DC}{KC} = \tan\theta, KC = \frac{2}{\tan\theta} \text{ m}$	1M 1A 1A <u>3</u>	any part in this question Wrong/no unit, pp-1. in the answer 2 + 1 in each part
(b) (i)	$S_1 = \frac{6}{2} (3 + 2)$ $= 15 \text{ m}^2$	1A	
(ii)	$S_2 = \frac{6}{2} \left( \frac{3}{\tan\theta} + \frac{2}{\tan\theta} \right)$ $= \frac{15}{\tan\theta} \text{ m}^2$	1A	
	$\therefore \frac{S_1}{S_2} = \frac{15}{\frac{15}{\tan\theta}} = \tan\theta$	1A	Must show working.
	$\frac{15}{\left(\frac{15}{\tan\theta}\right)} = \tan\theta$ $\tan\theta = \tan\theta$ 0 mark	<u>3</u>	$\frac{15}{\frac{15}{\tan\theta}} = \tan\theta$ = tan θ 1A (PP-1) = tan θ 1A (PP-1)



(c) Let $KE \perp BH$ .	
$EK = BC = 6 \text{ m}$	$K? = 6$ ——— 2 marks
$HE = \frac{3}{\tan \theta} - \frac{2}{\tan \theta} = \left( \frac{3}{\tan 30^\circ} - \frac{2}{\tan 30^\circ} \right) \text{ m} (= \sqrt{3})$	
$\therefore HK = \sqrt{HE^2 + EK^2}$	
$= \sqrt{(\sqrt{3})^2 + 6^2}$	
$= \sqrt{39} \text{ m} \dots\dots\dots$	

## Solutions

14. (a) (i)  $x^3 - \frac{4}{3}x - 6 = 0$  can be written as

$$x^3 = \frac{4}{3}x + 6.$$

Consider the line  $y = \frac{4}{3}x + 6$

It cuts the curve  $y = x^3$  at  $x = r$ ,

where  $r$  lies between 2.0 and 2.1.

(ii) Let  $f(x) = x^3 - \frac{4}{3}x - 6$

$$\left. \begin{array}{l} f(2) = - (= -0.67) \\ f(2.1) = + (= 0.46) \end{array} \right\} \begin{array}{l} \text{both correct} \\ \dots\dots\dots \end{array}$$

Interval	Mid-value $x$	$f(x)$
$2.000 < r < 2.100$	2.050 <sup>IM</sup>	- (= -0.12) <sup>1A</sup>
$2.050 < r < 2.100$	2.075	+ (= 0.17)
$2.050 < r < 2.075$	2.063	+ (= 0.02)
$2.050 < r < 2.063$	2.057	- (= -0.04)
$2.057 < r < 2.063$		

$\therefore r = 2.06$  (correct to 2 d.p.)

Alt. Solution:

$$f(2) = -$$

$$f(2.5) = +$$

) .....  
2.25 0M+0A

Interval	Mid-value $x$	$f(x)$
$2.000 < r < 2.500$	2.250	+
$2.000 < r < 2.225$	2.113	+
.	.	.
.	.	.
.	.	.

$\therefore r = 2.06$  (correct to 2 d.p.)

(b) Put  $x = t + 1$

The given equation can be written

$$\text{as } 3x^3 - 4x - 18 = 0$$

$$\text{or } x^3 - \frac{4}{3}x - 6 = 0$$

By (a), the solution is

$$t = 2.06 - 1 \dots\dots\dots$$

$$= 1.06 \text{ (correct to 2 d.p.)}$$

Marks

Remarks

1M

1A+1A

1A

1A for equation  
1A for line drawn,  
±1 vertical division  
about (0, 6), (3, 10)

1M

Correct change of sign.

1M+1A

1M

1M for choosing mid-  
value, 1A for correct  
sign.

Next correct <sup>interval</sup> step.

1A

9

1M

1M+1A

1M

1A

1A

1M

1A

3

Solutions

Marks

Remarks

14.

